reported gust field parameters: sample size, duration of primary data, size of variate, dropped variates, initial distributions, extremal distributions, mixed distributions, method of estimating parameters, etc.

Low-level gusts refer to gusts at low levels of altitude above the terrain, nominally less than 1000 ft, but may be at any pressure altitude depending on atmospheric conditions. Fits to the sample of 42 vertical gust velocity data runs analyzed in Ref. 4, for scale of turbulence and pressure altitude, both linear regression and probability density distribution, are as follows. The linear regression, p. 263 of Ref. 4, is

$$L = 467.827 + 45.274 \times 10^{-3}h \ldots \tag{4}$$

$$0 \leq h \leq 12{,}000 \text{ ft}$$

where L is the scale of turbulence, ft, and h is the pressure altitude, ft. The probability density distribution, p. 264 of Ref. 4, is

$$f(L) = \frac{1}{40.5 \times 10^6} L^3 \exp(-L/150)\ldots \tag{5}$$

where $f(L)$ is χ^2 distributed with mean 600, mode 450, median 550, and variance 90,000.

References

[1] Neuls, G. S., "Optimum fatigue spectra," Aeronautical Systems Division TDR 61-235 (August 1961).

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[3] USAF Military Specifications, MIL-A-8866, Aeronautical Standards Group (May 1960).

[4] Saunders, K. D., "B-66B low-level gust study," Vol. 1, *Technical Analysis*, Wright Air Development Division TR 60-305 (March 1961).

Optimum Spacing of Shell Frames

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Nomenclature

E_f = modulus of elasticity of frame, psi
I_f = moment of inertia of frame, in.^{4}
M = bending moment, in.-lb
R = radius of shell, in.
t_e = effective shell thickness, in.
L = frame spacing, in.
A_f = area of frame, in.^{2}
N = maximum axial loading, lb/in.
f_c = applied compressive stress, psi
F_c = allowable compressive stress, psi
γ_s = density of shell material, lb/in.^{3}
γ_f = density of frame material, lb/in.^{3}
k = shape factor
c = column fixity coefficient
K = shape factor

In Reference (1), Shanley develops an expression for the combined weight of covering and frame material per inch of fuselage length. He shows, graphically, that a frame spacing exists which yields the minimum weight of total structure. For many types of structures the allowable compressive stress may be approximated analytically in terms of loading; e.g., the curves in Fig. 1, illustrating structure manufactured in currently used materials, can be approximated by two straight lines. Using this approximation it is shown that the optimum frame spacing may be readily estimated.

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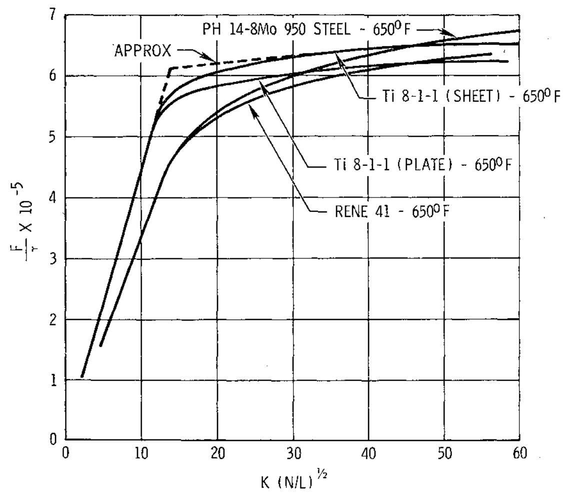

Fig. 1 Allowable compressive stress curves, approximated in terms of loading.

According to Ref. (1) the stiffness of the frame section required to prevent general instability is

$$I_f = \frac{4C_f R^2 M}{E_f L} \tag{1}$$

and

$$I_f = kA_f^2 \tag{2}$$

Substituting A_f from Eq. (2) into Eq. (1), solving for A_f, and dividing by L,

$$\frac{A_f}{L} = \frac{2RM^{1/2}}{L^{3/2}}\left(\frac{C_f}{kE_f}\right)^{1/2} \tag{3}$$

Then the combined weight of the shell and frame per inch of fuselage length may be written

$$W = 2\pi R t_e \gamma_s + \frac{4\pi R^2 M^{1/2}\gamma_f}{L^{3/2}}\left(\frac{C_f}{kE_f}\right)^{1/2} \tag{4}$$

Now

$$N = f_c t_e = \frac{M}{\pi R^2} \tag{5}$$

Solving for t_e from Eq. (5) and substituting into Eq. (4),

$$W = \frac{2\pi R N \gamma_s}{f_c} + \frac{4\pi^{3/2}R^3 C_f c^{3/4}\gamma_f}{NkE_f^{1/2}}(N/L')^{3/2} \tag{6}$$

where $L' = L/(c)^{1/2}$ The allowable compressive stress in the surface can be approximated by a linear function of $(N/L')^{1/2}$:

$$\begin{aligned} F_c &= A(N/L')^{1/2} \qquad (N/L')^{1/2} < \psi \\ F_c &= B + H(N/L')^{1/2} \qquad (N/L')^{1/2} > \psi \end{aligned} \tag{7}$$

Substituting F_c from Eq. (7) into Eq. (6), we obtain

$$W = \frac{2\pi R N \gamma_s}{B + H(N/L')^{1/2}} + \frac{4\pi^{3/2}R^3 C_f^{1/2}\gamma_f}{k^{1/2}c^{3/4}E_f^{1/2}}(N/L')^{3/2} \tag{8}$$

Letting $\alpha = (N/L')^{1/2}$, Eq. (8) becomes

$$W = [J/(B + H\alpha)] + P\alpha^3 \tag{9}$$

The value of α corresponding to the minimum value of W is obtained by differentiating W with regard to α, setting equal to zero, and solving for α:

$$\frac{dW}{d\alpha} = -\frac{JH}{(B + H\alpha)^2} + 3P\alpha^2 \tag{10}$$

$$\alpha_{\text{opt}} = \frac{-B \pm [B^2 + 4H\,(JH/3P)^{1/2}]^{1/2}}{2H} \tag{11}$$

$$J = 2\pi RN\gamma_s \qquad P = \frac{4\pi^{3/2}R^3C_f{}^{1/2}\gamma_f}{k^{1/2}c^{3/4}NE_f{}^{1/2}}$$

Reference

[1] Shanley, F. R., *Weight-Strength Analysis of Aircraft Structures* (McGraw-Hill Book Co., Inc., New York, 1952), pp. 73–81.

Buckling of Shell-Supported Rings

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Nomenclature

A = cross-sectional area of ring, in.^{2}
B = point of support
C = centroid
E = modulus of elasticity, psi
G = shear modulus, psi
I_x, I_y = moment of inertia about x and y directions, respectively, in.^{4}
I_p = polar moment inertia about z axis, in.^{4}
I_{xy} = product of inertia, in.^{4}
J = torsion constant (St. Venant), in.^{6}
M = moment resultant, in.-lb/in.
N = direct stress resultant, lb/in.
S = shear center
U = internal strain energy, in.-lb
V = potential energy, in.-lb
W = work done by external forces, in.-lb
k_x, k_y, k_z = spring constant of support in x, y, and z directions, lb/in./in.
k_β = rotational spring, lb/rad/in.
m = integer
p = external pressure, subscript for polar, psi
q = external line of pressure, lb/in.
r, R = radius, in.
u, v, w = displacement component in x, y, z directions, respectively, in.
x, y, z = coordinate directions and dimensions, in.
α = half-apex angle of a conical support, rad
β = rotation of cross section, rad
Γ = warping constant
ϵ = strain, in./in.
ϕ = angular coordinate ($z = r_s\phi$), rad
θ = twist, rad/in.
$\kappa_x, \kappa_y, \kappa_z$ = curvature change associated with moments about the x, y, and z coordinates, respectively, 1/in.
ρ = $x + y \tan\alpha$, in.

Introduction

The purpose of this note is to develop ring formulas that will be useful in design and analysis of ring-stiffened shells. A ring stiffener is not a free ring for it is constrained by the attached shell. A first approximation of the shell constraint is to assume that the ring is rigidly supported in the direction of the shell meridian and has elastic support from the shell in the radial and tangential ring directions and in rotation.

The ring is loaded by an external line of pressure q which arises from the ring-shell discontinuity analysis.

The coordinates are x, measured inward from the shear center; y, normal to the plane of the ring; and $z = r_s\phi$ along the axis of shear centers. Corresponding displacements are u, v, and w, plus a rotation of the cross section β.

The constraint of the shell in the meridional direction requires that

$$v = (u - y_b\beta)\tan\alpha - x_b\beta \tag{1}$$

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where α is the half-apex angle of a cone tangent to the middle surface of the shell at the point of attachment of the ring. Figure 1 indicates the geometry of this support. It is always possible to represent the elastic support by a set of springs k_x, k_β, and k_z attached at x_b, $y_b = y_c$. The external line of pressure q is applied at this same point in order that the prebuckling stress distribution be free of primary bending stress. The total potential energy expression with the constraint of Eq. (1) becomes[1]

$$\begin{aligned} V = \tfrac{1}{2}\int \Big\{ & EA\epsilon_0{}^2 + EI_x\kappa_x{}^2 + EI_y\kappa_y{}^2 - 2EI_{xy}\kappa_x\kappa_y + GJ\theta^2 + \frac{E\Gamma}{r_s{}^2}\theta'^2 + \Big[k_x(u - y_c\beta)^2 + k_\beta\beta^2 + k_z \times \\ & \left(w\frac{r_b}{r_s} - \frac{x_bu'}{r_s} - \frac{y_c}{r_s}(u'\tan\alpha - \rho_b\beta')\right)^2\Big]\frac{r_b}{r_c} - \frac{q}{r_c} \times \\ & \Big[u'^2 + (u'\tan\alpha - \rho_b\beta')^2 - u^2 - 2y_c(u'\beta' - u\beta) + 2x_c \times \\ & (u'\tan\alpha - \rho_b\beta')\beta' + \frac{I_p}{A}\beta'^2 - \frac{(I_x + y_c{}^2A)}{A}\beta^2 - r_c \times \\ & (x_c - x_b\beta^2\Big]\Big\} r_c d\phi \end{aligned} \tag{2}$$

where

$$\epsilon_0 = \frac{w'}{r_s} - \frac{\rho_c}{r_cr_s}u'' - \frac{u}{r_c} + \frac{y_c\rho_b}{r_cr_s}\beta'' + \frac{y_c}{r_c}\beta$$

$$\kappa_x = \frac{\beta}{r_c} - \frac{(u''\tan\alpha - \rho_b\beta'')}{r_cr_s}$$

$$\kappa_y = \frac{u''}{r_c{}^2} + \frac{y_c}{r_c{}^2r}(u''\tan\alpha - \rho_b\beta'') + \frac{u}{r_c{}^2} - \frac{y_c}{r_c{}^2}\beta$$

$$\theta = \frac{\beta'}{r_s} + \frac{(u'\tan\alpha - \rho_b\beta')}{r_s{}^2}$$

$$\rho_b = x_b + y_c\tan\alpha$$

$$\rho_c = x_c + y_c\tan\alpha$$

The procedure is to set the first variation of the potential energy equal to zero. The fundamental lemma of the calculus of variations leads to the equations of equilibrium. A solution to the equations of equilibrium of the form,

$$u = u_0\sin m\phi \tag{3}$$

$$w = w_0\cos m\phi \tag{4}$$

$$\beta = \beta_0\sin m\phi \tag{5}$$

may be assumed, which satisfies the requirement in a ring that the displacements be periodic on ϕ. Upon substitution of

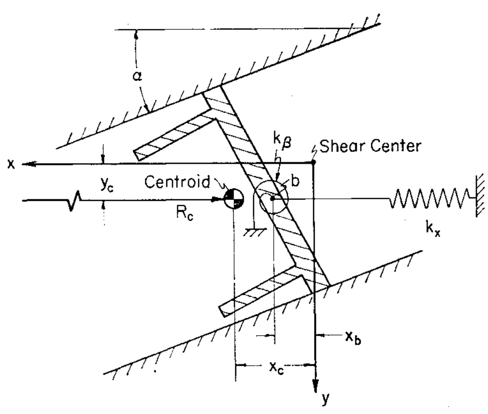

Fig. 1 Coordinates used and constraints approximating the supporting shell.